CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet I
November 8, 2000 **Category III (Advanced)**

Miscellaneous Problems (point values as indicated). On your answer sheet, circle the correct response or write your answer in the blank(s) provided. (P) means that partial credit may be given.

1. [2 points each](P) *True/False: On your answer sheet, circle "T" for each of the following statements which is always true; circle "F" for each statement which is not always true. Assume a, b, c, u, and v are all real numbers.*

   (a) $\sqrt{a^2+b^2} = a+b$

   (b) $a^2a^3 = a^6$

   (c) If $f(x) = \dfrac{1}{x}$, then $f(x+h) = \dfrac{1}{x} + \dfrac{1}{h}$.

   (d) If $u + v\sqrt{-1}$ is a root of ax^2+bx+c, then $u - v\sqrt{-1}$ is also a root.

   (e) If $a^3 < b^3$, then $a \leqslant b$.

2. [2 points each](P) *True/False: On your answer sheet, circle "T" for each of the following statements which is always true; circle "F" for each statement which is not always true.*

   (a) $0.14 \geqslant \dfrac{1}{7}$

   (b) $0.1428 \geqslant \dfrac{1}{7}$

   (c) $0.142857 \geqslant \dfrac{1}{7}$

   (d) $0.\overline{142857} \geqslant \dfrac{1}{7}$

   (e) $\dfrac{1}{6} \geqslant \dfrac{1}{7}$

3. [10 points] Express $\dfrac{1}{\left(\sqrt{2}\right)^3 + \sqrt{2} + 1}$ in the form $a + b\sqrt{2}$ where a and b are rational.

4. [10 points] Find the y-coordinate of the minimum point on the graph of $f(x) = 3(x-1)^2 + 5$.

5. [10 points] The mean of the numbers 3, 7, 10, 12, -3, 5, and x is 5. Find x.

6. [10 points] Find the *exact* solution to the system of equations. If there is no solution, write *none* in one of the blanks on your answer sheet.

$$\begin{aligned} x - 4y &= 1 \\ 2x - y &= -3 \\ -x - 3y &= 4 \end{aligned}$$

7. [10 points] Find the *exact* coefficient of a in $f\left(a + \sqrt{2}\right) - f(a)$ for $f(x) = 2x^2 - 3x$.

8. [10 points] Find the smallest integer n such that $1.96\dfrac{30}{\sqrt{n}} \leqslant 4.21$.

9. [10 points] In a high school with a population of 1000, a group of students spread the rumor "Participate in the Mathematics League contest, and you will become rich and famous." The number of people $N(t)$ in the high school who have heard the rumor after t minutes is given by:

$$N(t) = \frac{1000}{1 + 132e^{-0.4t}}$$

After how many minutes will half of the high school population have heard the rumor? *Round your answer to the nearest tenth of a minute.*

10. [10 points] Find the value(s) of x which solve the following inequality. *Write your answer as an inequality.*

$$|x-1| + |x-2| + |x-3| + |x-4| + |x-5| + |x-6| + |x-7| + |x-8| \leqslant 16$$

Student's Answer Sheet

Name: _____
PRINT: First Last

School: _____ Code [ ]

Miscellaneous Problems (point values as indicated). Circle the correct response or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given.

1. (a) **T** **F** — 10P [ ]
   (b) **T** **F**
   (c) **T** **F**
   (d) **T** **F**
   (e) **T** **F**

2. (a) **T** **F** — 10P [ ]
   (b) **T** **F**
   (c) **T** **F**
   (d) **T** **F**
   (e) **T** **F**

3. $a + b\sqrt{2} =$ _____ + _____ $\sqrt{2}$ — 10 [ ]

4. y-coordinate = _____ — 10 [ ]

5. mean $x =$ _____ — 10 [ ]

6. $x =$ _____ $y =$ _____ — 10 [ ]

7. coefficient of $a =$ _____ — 10 [ ]

8. smallest integer $n =$ _____ — 10 [ ]

9. $t \approx$ _____ minutes — 10 [ ]

10. x satisfies _____ — 10 [ ]

TOTAL SCORE [ ]