CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet I
November 8, 2000

ANSWER KEYS

Category I (Geometry)

1. **b** 2. **b** 3. **d** 4. **e** 5. **e**

6. **c** 7. **a** 8. **e** 9. **c** 10. **b**

11. If x represents the measure of angle X, then $90^\circ - x = \frac{4}{9}(180^\circ - x)$. Solving for x results in $x = 18^\circ$.

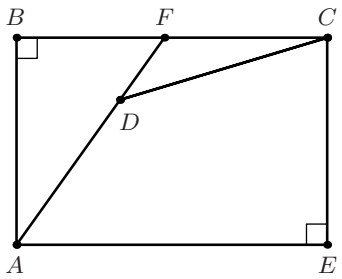
12. Each of the five points can be paired with four others to give four different lines because no three points are collinear. Since each of these 20 lines was counted exactly twice, there are a total of 10 distinct lines.

13. Using the large triangle with the 71° angle, we have $a = 180^\circ - 40^\circ - 71^\circ = 69^\circ$. Because of supplementary angles, $b = 180^\circ - 133^\circ = 47^\circ$. Also, $180^\circ - c = 180^\circ - a - b = 180^\circ - 69^\circ - 47^\circ = 64^\circ$ yields $c = 116^\circ$, and $d = 180^\circ - (180^\circ - 140^\circ) - 47^\circ$ yields $d = 93^\circ$. Because of vertical angles, the angle measures in the small triangle at the top are b, $180^\circ - 40^\circ - d$, and e. Since they sum to 180°, it follows that $e = 86^\circ$.

14. Each hour corresponds to an angle of $360^\circ/12 = 30^\circ$. If h is the number of hours which have elapsed since the clock read 12 o'clock, then $\frac{h}{1} = \frac{57^\circ}{30^\circ} = 1.9$. Since 0.9 hours equals 54 minutes, the clock will read 1:54.

15. First note that $m(\angle DEC) = m(\angle DCE)$ since $DE = DC$. Also, $m(\angle EDC) = 180^\circ - m(\angle ADE) = 180^\circ - 60^\circ = 120^\circ$. If x represents the measure of $\angle DCE$, then $2x + 120^\circ = 180^\circ$ and thus $x = 30^\circ$.

16. Form a point E so that the convex polygon $ABCE$ is a rectangle (see figure). Then $m(\angle DCE) = 90^\circ - 17^\circ = 73^\circ$. Let $x = m(\angle DAB)$. Since the sum of the interior angles of a convex polygon with n sides equals $(n-2)180^\circ$, the sum of the interior angles of the convex quadrilateral $ADCE$ equals 360°. Therefore, $90^\circ - x + 129^\circ + 73^\circ + 90^\circ = 360^\circ$. Solving for x gives $x = 22^\circ$. Alternatively, if $\overline{AD}$ is extended to $\overline{BC}$ at point F, then $m(\angle CDF) = 180^\circ - 129^\circ = 51^\circ$ and so $m(\angle BFA) = 180^\circ - (180^\circ - (51^\circ + 17^\circ)) = 68^\circ$. Hence $x = 90^\circ - 68^\circ = 22^\circ$.



Category II (Algebra)

1. (a) True (b) True (c) False (d) True (e) False

2. **a** 3. **d** 4. **e** 5. **a**

6. **d** 7. **b** 8. **e** 9. **c**

10. There are a total of 81 two-digit numbers formed from the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$. Out of these, 9 have identical units and tens digits. Since one half of the remaining numbers have their units digit greater than their tens digit, the answer is $72/2 = 36$.

11. Rewriting $x - 3y = 1$ yields $y = \frac{1}{3}x - \frac{1}{3}$. Since the two lines are perpendicular, it follows that $m = -1/(\frac{1}{3}) = -3$. Hence $y - 1 = -3(x - 5)$ and thus $y = -3x + 16$.

12. Let x equal the number of employees who own a bike. Then $x = 0.4(x + 66 - 21)$. Solving, we get $x = 30$. Therefore, there are $30 - 21 = 9$ employees who own a bike but not a pet.

13. Since $f\left(\frac{1}{2}\right) = \dfrac{1}{\frac{1}{2} - 1} = -2$, it follows that $g\big(f(\frac{1}{2}) + 1\big) = g(-2 + 1) = g(-1) = \dfrac{1 + (-1)}{1 - (-1)} = 0$.

14. Let s, m, and l represent the number of small, medium, and large pizzas, respectively, made in one day. Then $2s + 3m + 4l = 64$, $s + m + 2l = 27$, and $4s + 5m + 6l = 108$. This system of linear equations has the solution $s = 7$, $m = 10$, and $l = 5$. Therefore, they make 7 small pizzas in one day.

Category III (Advanced)

1. (a) False (b) False (c) False (d) True (e) True

2. (a) False (b) False (c) False (d) True (e) True

3. Simplify and rationalize the denominator to get $a = -1/17$ and $b = 3/17$.

4. $f(x)$ is clearly the function $g(x) = x^2$ shifted 1 unit right and 5 units up. Thus, it has a minimum value of 5.

5. The definition of the mean says that $5 = \dfrac{34 + x}{7}$; solve to get $x = 1$.

6. Solve any two of the equations to find $x = -13/7$ and $y = -5/7$. This solution works in the third equation. Thus, it is the answer.

7. Expand to get $f(a + \sqrt{2}) - f(a) = 4\sqrt{2}a + 4 - 3\sqrt{2}$. Thus, the coefficient of a is $4\sqrt{2}$.

8. Solve the inequality to get $n \geqslant \left(\dfrac{(1.96)(30)}{4.21}\right)^2$ or $n \geqslant 195.07$. Thus, the answer is 196.

9. Set $N(t) = 500$ and solve. Simplify to get $132e^{-.4t} = 1$, or $t = \dfrac{\ln(132)}{.4} \approx 12.2$ minutes.

10. Either graph the two sides of the inequality on a calculator and find the inequality is satisfied when $4 \leqslant x \leqslant 5$. Or, notice that the function on the left is a piecewise linear function and sketch the graph by plotting the points when $x = 1, 2, 3, 4, 5, 6, 7$ and 8. This sketch will show that the piecewise line has slope 0 and value 16 on the interval $4 \leqslant x \leqslant 5$. (Note: If you replace the 1, 2, 3, 4, 5, 6, 7, 8 with any data values, the resulting function is minimized at the median of the data values.)