CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet II
January 30, 2001

Category I (Geometry)

Miscellaneous Problems (point values as indicated). On your answer sheet, circle the correct response or write your answer in the blank(s) provided. (P) means that partial credit may be given. Figures are not necessarily drawn to scale. Unless otherwise noted, all questions refer to Euclidean plane geometry.

1. [2 points each](P) *True/False: On your answer sheet, circle "T" for each of the following statements which is always true; circle "F" for each statement which is not always true.*

   (a) All equilateral triangles are congruent.

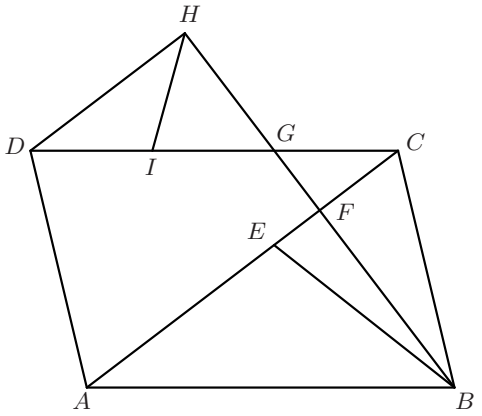
   (b) A line segment has only one bisecting line.

   (c) The measure of an interior angle of a regular polygon with $2n$ sides is twice the measure of an interior angle of a regular polygon with n sides.

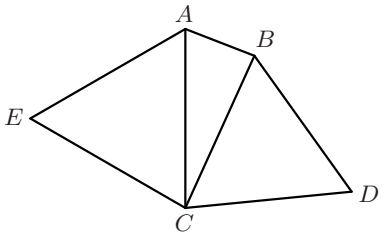
   (d) A trapezoid is a quadrilateral with at least two parallel sides.

   (e) If a base angle of an isosceles triangle has measure less than 60°, then the base is the shortest side of the triangle.

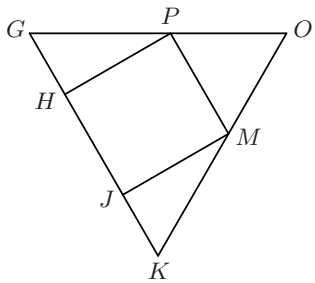
2. [20 points](P) Quadrilateral $ABCD$ is a parallelogram, $\overline{AC} \parallel \overline{DH}$, $\overline{BE}$ bisects $\angle ABC$, $\overline{BF}$ is an altitude of $\triangle ABC$, and $\overline{HI}$ is a median of $\triangle GHD$. If $m(\angle CAB) = 27^\circ$ and $m(\angle BCA) = 79^\circ$, find the *exact* measures of $\angle ABE$, $\angle CDA$, $\angle EBF$, $\angle IGH$, and $\angle GHI$.



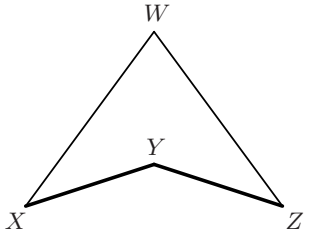
3. [10 points](P) Triangles BCD and AEC are equilateral. Using the points A, B, C, D, and E, find an angle different from $\angle ECB$ which *must* be congruent to $\angle ECB$ and find a segment different from $\overline{AD}$ which *must* be congruent to $\overline{AD}$.



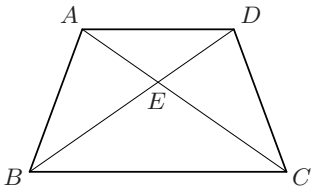
4. [10 points](P) Rectangle $PHJM$ is inscribed in $\triangle GKO$, $GH = 1.28$ inches, $KJ = 1.28$ inches, and $m(\angle KOG) = 110^\circ$. Find the *exact* measures of $\angle OPM$ and $\angle KMJ$.



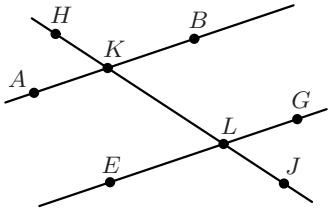
5. [10 points] Points X, Y, and Z are three consecutive vertices of a regular n-gon with $n \geqslant 7$ such that Y is between X and Z. Point W lies in the exterior of the n-gon such that $m(\angle WXY) = 60^\circ = m(\angle WZY)$. Find the *exact* measure of $\angle XWZ$ when $n = 36$.



6. [10 points] An isosceles trapezoid $ABCD$ has congruent legs $\overline{AB}$ and $\overline{DC}$. If the diagonals of the trapezoid meet at E, $AE = x + 7$, $EC = 3x - 1$, and $DB = 26$, find the *exact* value of DE.



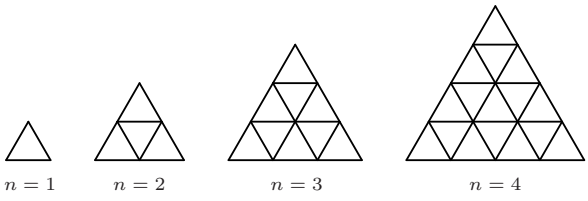
7. [10 points] If $\overleftrightarrow{AB}$ and $\overleftrightarrow{EG}$ are parallel, angle BKL has measure $9x - 13$, and $m(\angle BKH) = 21x + 6$, find the *exact* measure of $\angle ELJ$.



8. [10 points] A rectangle is divided into four rectangles with areas $\sqrt{18}$, $\sqrt{3}$, $\sqrt{2}$, and x as shown in the figure. Find the *exact* value of x.

$\sqrt{18}$	$\sqrt{3}$
x	$\sqrt{2}$

9. [10 points] Consider the grids of small equilateral triangles for $n = 1, 2, 3, 4$ shown in the figure. Note that the total number of distinct equilateral triangles of all sizes and orientations equals five when $n = 2$. Find the total number of distinct equilateral triangles of all sizes and orientations in the grid for $n = 4$.



Student's Answer Sheet

Name: _____
PRINT: First Last

School: _____ Code ☐

I participated in Meet I: Yes ☐ No ☐

Miscellaneous Problems (point values as indicated). Circle the correct response or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given.

1. (a) **T** **F** — 10P ☐
   (b) **T** **F**
   (c) **T** **F**
   (d) **T** **F**
   (e) **T** **F**

2. $m(\angle ABE) =$ _____ $^\circ$ $m(\angle CDA) =$ _____ $^\circ$ $m(\angle EBF) =$ _____ $^\circ$ — 20P ☐

   $m(\angle IGH) =$ _____ $^\circ$ $m(\angle GHI) =$ _____ $^\circ$

3. angle _____ $\cong \angle ECB$ segment _____ $\cong \overline{AD}$ — 10P ☐

4. measure of $\angle OPM =$ _____ $^\circ$ measure of $\angle KMJ =$ _____ $^\circ$ — 10P ☐

5. measure of $\angle XWZ =$ _____ $^\circ$ — 10 ☐

6. $DE =$ _____ — 10 ☐

7. measure of $\angle ELJ =$ _____ $^\circ$ — 10 ☐

8. $x =$ _____ — 10 ☐

9. total number of triangles = _____ — 10 ☐

TOTAL SCORE ☐