CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet II
January 30, 2001

ANSWER KEYS

Category I (Geometry)

1. (a) False (b) False (c) False (d) False (e) False

2. $m(\angle ABE) = 37^\circ$, $m(\angle CDA) = 74^\circ$, $m(\angle EBF) = 26^\circ$, $m(\angle IGH) = 63^\circ$, and $m(\angle GHI) = 63^\circ$.

3. Since $\triangle ECB \cong \triangle ACD$ by SAS, it follows that $\angle ECB \cong \angle ACD$ and $\overline{AD} \cong \overline{EB}$.

4. Since $\triangle GHP \cong \triangle KJM$ by SAS and because $\square PHJM$ is a rectangle, it follows that $m(\angle OPM) = m(\angle PGH) = m(\angle MKJ) = (180^\circ - 110^\circ)/2 = 35^\circ$. Then $m(\angle KMJ) = 90^\circ - m(\angle MKJ) = 55^\circ$.

5. The measure of each interior angle of a regular n-gon is $180^\circ(1 - \frac{2}{n})$; thus $m(\angle XYZ) = 180^\circ(1 - \frac{2}{n})$. In isosceles triangle XZY, let $x = m(\angle YXZ) = m(\angle YZX)$. Then $2x + 180^\circ(1 - \frac{2}{n}) = 180^\circ$ and so $x = \frac{180^\circ}{n}$. Because $2(x+60^\circ)+m(\angle XWZ) = 180^\circ$, it follows that $m(\angle XWZ) = 60^\circ(1-\frac{6}{n})$. Hence $m(\angle XWZ) = 50^\circ$ when $n = 36$.

6. Since $\overline{AE} \cong \overline{DE}$ and $\overline{EC} \cong \overline{EB}$, it follows that $AE + EC = x + 7 + 3x - 1 = 26 = DB$ and so $x = 5$. Thus $DE = AE = x + 7 = 12$.

7. Angles BKL and BKH are supplementary; thus $9x - 13 + 21x + 6 = 180$ and so $x = \frac{187}{30}$. Because the alternate exterior angles BKH and ELJ are congruent, it follows that $m(\angle ELJ) = (21x + 6)^\circ = \left(\frac{1369}{10}\right)^\circ = 136.9^\circ$.

8. If the rectangle has dimensions $a + b$ by $c + d$ (see figure), then $ac = \sqrt{18}$, $bc = \sqrt{3}$, $ad = x$, and $bd = \sqrt{2}$. Hence

$$x = ad = \frac{\sqrt{18}}{c} \cdot \frac{\sqrt{2}}{b} = \frac{\sqrt{18}\sqrt{2}}{bc} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

	a	b
c	$\sqrt{18}$	$\sqrt{3}$
d	x	$\sqrt{2}$

9. There are 27 triangles when $n = 4$. In general, there are $(4n^3 + 10n^2 + 4n - 1 + (-1)^n)/16$ triangles in the grid for $n = 1, 2, \ldots$.

Category II (Algebra)

1. (a) False (b) True (c) False (d) True (e) True

2. **d** 3. **d** 4. **c** 5. **b**

6. **e** 7. **c** 8. **d** 9. **b**

10. The maximum number of square feet corresponds to a circular play area. If r denotes the radius, then setting $160 = 2\pi r$ gives $r = 80/\pi$. Hence the maximum area is $\pi r^2 = 80^2/\pi \approx 2037$ square feet.

11. If x denotes the monthly rent in dollars, then the repair/maintenance account contains $(12)(14.5\%)x$ dollars. Solving for the smallest x such that $12x - (12)(14.5\%)x \geqslant \$9250$ gives $x \approx \$902$.

12. If x, y, and z denote the dollar amounts invested at 7%, 8%, and 9%, respectively, then $x + y + z = 2500$, $.07x + .08y + .09z = 212$, and $z - y = 1100$. The solution of this system is $x = 400$, $y = 500$, and $z = 1600$. Thus \$500 is invested at 8%.

13. If p and f denote the number of students who passed and the number who failed, respectively, then $p + f = 30$ and $75p + 45f = (30)(68)$. The solution of this systen is $p = 23$ and $f = 7$. Therefore 23 students passed the exam.

14. If n denotes the number of dimes, then $n+1$ is divisible by 3, 4, and 5. Therefore $n+1$ must be divisible by 60, which is the least common multiple of 3, 4, and 5. Since $10 < n < 100$, it follows that $n = 59$.

Category III (Advanced)

1. From the statement of the problem, we know x and y are integers between 0 and 99, inclusive, and that $y - x = 18$. (a) is true because $x > 90$ would make $y > 108$. (b) is true because $y = 36$ and $x = 18$ is a possible solution. (c) is true because $x = 22$, $y = 40$ is a solution which makes the amount of the check \$22.40. (d) is false since this additional fact forces $x = 1764/101$, which is not an integer. (e) is false because $x = 22$, $y = 40$ is a solution with the sum of digits = 8.

2. Square both sides, subtract $\sqrt{x}$ from both sides, square again, then simplify and combine terms to get $18\sqrt{x} = 78$. Divide by 18 and square one more time to get $x = 169/9$. This answer checks.

3. Multiply both sides by $x(2x-1)$ to get $2x^2 + 2x - 1 = x$; combine terms and solve to get $x = 1/2$ or $x = -1$. Since the $x = 1/2$ root is extraneous, the solution is $x = -1$.

4. Using properties of logarithms, it follows that $\log_5\left(7^{\log_7 x}\right) = \log_5 x = 3$. Therefore $x = 5^3 = 125$.

5. Adding the first two equations gives $-3y = 2$ and so $y = -2/3$. By replacing y with this value in the second equation, we obtain $x = -11/6$. However, adding the second two equations gives $2x = 6$ and so $x = 3$. The system of equations has no solution.

6. The game which costs 14300 Mexican centavos costs $(14300)(1/100)(1/9.59) \approx 14.91$ US Dollars. The game which costs 17.5 Euros costs $(17.5)(0.848536) \approx 14.85$ US Dollars. The least expensive item costs 14.85 US Dollars.

7. R changes by $\dfrac{1}{\frac{1}{83} + \frac{1}{105}} - \dfrac{1}{\frac{1}{80} + \frac{1}{100}} = +\dfrac{3235}{1692} \approx +1.9$. Hence R increases by about 1.9 ohms.

8. If x represents the desired height, then by using the 45° and 30° triangles we can say $\dfrac{300 + x}{x} = \sqrt{3}$ and so $x = \dfrac{300}{\sqrt{3} - 1} \approx 410$ feet.

9. Completing the square, we have $f(x) = a\left(x - \dfrac{1}{3a}\right)^2 + 7 - \dfrac{1}{9a}$. Setting $7 - \dfrac{1}{9a} = 85$ gives $a = \dfrac{-1}{702}$.

10. The polynomial $x^{15} + ax + b$ is divisible by $x^2 - 1$ precisely when 1 and -1 are roots of $x^{15} + ax + b$. Hence $1 + a + b = 0$ and $-1 - a + b = 0$. This system of equations has the solution $a = -1$ and $b = 0$.