CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet III
March 29, 2001

Category III (Advanced)

Miscellaneous Problems (point values as indicated). On your answer sheet, circle the correct response or write your answer in the blank(s) provided. (P) means that partial credit may be given. Unless otherwise specified, assume all variables represent real numbers.

1. [2 points each](P) *True/False: On your answer sheet, circle "T" for each of the following statements which is always true; circle "F" for each statement which is not always true.*

   (a) $\log_a b \cdot \log_b a = 1$ for all $a > 0$, $b > 0$, $a \neq 1$, and $b \neq 1$.

   (b) The graph of the solutions to the equation $4x^2 - 16x = 2y^2 - 8$ is a hyperbola.

   (c) $\sin^2 x - \cos^2 x = \cos 2x$ for all x.

   (d) In the complex numbers, $(3 + 2i)^2 = 5$.

   (e) The three terms $\sin x$, $1 - \cos^2 x$, $\sin^3 x$ form three consecutive terms in a geometric sequence.

2. [5 points] If $\log_{10} m = b - \log_{10} n$, then $m =$

   a. $\dfrac{b}{n}$

   b. bn

   c. $10^b n$

   d. $\dfrac{10^b}{n}$

   e. $\dfrac{b}{10^n}$

3. [5 points] If $f(x) = 2x^2 - 3x$, which one of the following is equal to $\dfrac{f(x+h) - f(x)}{h}$ for $h \neq 0$?

   a. $4x + 2h - 3$

   b. $2x + h - 3$

   c. $\dfrac{4xh + 2h^2 - 6x - 3h}{h}$

   d. $\dfrac{2xh + h^2 - 6x - 3h}{h}$

   e. $4xh + 2h - 6x - 3$

4. [10 points] Approximate $\sqrt{2}^{3^{\sqrt{2}^3}}$. *Round your answer to the nearest integer.*

5. [10 points](P) If $f(x) = ab^x$, $f(1) = 4$, and $f(2) = 9$, find a and b. *Give exact answers for both a and b.*

6. [10 points] Find the positive real solution to $\left(x^2 + 2^x + x2^{\frac{x+1}{2}}\right)\left(x^2 + 2^x - x2^{\frac{x+1}{2}}\right) - 4^x - 25 = 0$. *Give the exact answer.*

7. [10 points] Solve $\sin^{2000} x - \cos^{2000} x = 1$ for $0 \leqslant x \leqslant \pi$. *Give exact answer(s).*

8. [10 points] Five players play a simple card game. Ten cards numbered 1 to 10 are shuffled and randomly distributed two to each player. Each player totals the numbers on his/her two cards and the player with the highest point total wins. If two or more players have the same high total, the player with the highest card wins. For example, if Player 1 has a 5 and 7, and Player 2 has an 8 and 4, and 12 is the highest total, then Player 2 would win because he/she has the highest card in a winning hand. If you are Player 1, find the probability that you win the first game. *Give your answer as a common fraction reduced to lowest terms.*

9. [10 points] A particle is placed on the parabola $y = x^2 - x - 6$ at a point P with coordinates $(x_1, 6)$. The particle moves along the parabola until it reaches the other point Q with coordinates $(x_2, 6)$. Find the horizontal distance traveled by the particle, i.e. find $|x_1 - x_2|$. *Give the exact answer.*

10. [10 points] The seasonal variation in the length of daylight may be represented by a sine curve. New Orleans has about 14 hours of daylight at midsummer (the high point on the curve) and about 9 hours 20 minutes of daylight 6 months later at midwinter (the low point on the curve). If the sine curve model is used with the assumption that all months have equal length, how many hours of daylight would there be 1 month after midsummer? *Round your answer to the nearest minute.*

11. [10 points] The angle of inclination of an agricultural grain conveyor can be set between 20° and 50°. If the length of the conveyor is 8 meters, what is the <u>difference</u> between the maximum and minimum heights the conveyor can reach? *Round your answer to the nearest tenth of a meter.*

Student's Answer Sheet

Name: _____
PRINT: First Last

School: _____ Code ☐

I participated in: Meet I ☐ Meet II ☐ Neither ☐

Miscellaneous Problems (point values as indicated). Circle the correct response or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given.

1. (a) **T** **F** — 10P ☐
   (b) **T** **F**
   (c) **T** **F**
   (d) **T** **F**
   (e) **T** **F**

2. circle one: **a** **b** **c** **d** **e** — 5 ☐

3. circle one: **a** **b** **c** **d** **e** — 5 ☐

4. nearest integer = _____ — 10 ☐

5. $a =$ _____ $b =$ _____ — 10P ☐

6. $x =$ _____ — 10 ☐

7. solution(s) for x is/are: _____ — 10 ☐

8. probability = _____ — 10 ☐

9. difference = _____ meters — 10 ☐

10. _____ hours and _____ minutes — 10 ☐

11. _____ meters — 10 ☐

TOTAL SCORE ☐