INSTRUCTIONS

1. You will have 55 minutes to complete the exam. A 5-minute warning will be given after 50 minutes.

2. On the answer sheet provided, PRINT your name and school name where indicated, and optionally print your email address and high school graduation year. Mark the proper box to indicate whether you are an alternate for this meet. All answers are to be transferred to the answer sheet, which will be the only page collected. The exam itself is yours to keep and may be marked in any way you wish.

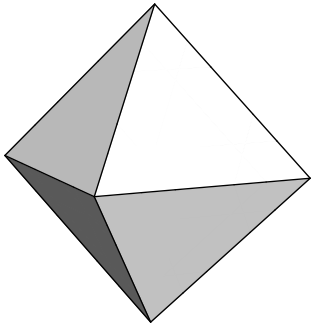
3. Answers will be graded all right or all wrong, except that partial credit may be given for those problems, if any, marked (P). Unless otherwise specified, all answers should be exact and written in simplest form.

4. No calculators may be used. Cell phones and music players are to be turned off and stowed away out of sight.

CENTRAL WISCONSIN

Mathematics League

Unplugged



Algebra, No Calculators

Category II

Meet I November 20, 2019

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CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet I November 20, 2019

Category II (Algebra, No Calculators)

Unless otherwise noted, all constants and variables represent real numbers.
Unless otherwise specified, all answers should be exact and written in simplest form.

1–6: Multiple Choice (6 points each). On your answer sheet, mark **X** *in the box for the one best choice.*

1. Which of the following is a solution to the equation $x^2 - 6x + 1 = 0$?

   a. $3 + 4\sqrt{2}$

   b. $-3 - 2\sqrt{2}$

   c. $-3 + 2\sqrt{2}$

   d. $3 - 2\sqrt{2}$

   e. $-3 - 4\sqrt{2}$

2. Once multiplied out, the product $2^{42} \cdot 5^{38}$ has how many digits?

   a. thirty-eight

   b. forty

   c. forty-two

   d. sixty-four

   e. eighty

3. Simplify: $\dfrac{x - 3(x-2)}{x-3}$

   a. $x - 2$

   b. 2

   c. $\dfrac{x^2 - 5x + 6}{x - 3}$

   d. -2

   e. $x - 3$

4. Which of the following integers has the greatest value?

   a. 5^{55}

   b. $\left(5^5\right)^5$

   c. $5^{(5^5)}$

   d. $\left(5^5\right)\left(5^5\right)$

   e. 55^5

5. Evaluate: $\dfrac{2}{\frac{1}{3} + \frac{1}{4}}$

   a. $\dfrac{24}{7}$

   b. $\dfrac{4}{7}$

   c. 14

   d. $\dfrac{7}{4}$

   e. $\dfrac{7}{24}$

6. Numberphile Andrew Booker recently found three integers x, y, z so that $x^3 + y^3 + z^3 = 42$. He showed that

$$42 = (-80{,}538{,}738{,}812{,}075{,}974)^3 + (80{,}435{,}758{,}145{,}817{,}515)^3 + z^3$$

   when z equals one of the following integers. Which of the integers equals z?

   a. 12,602,123,297,335,628

   b. 12,602,123,297,335,629

   c. 12,602,123,297,335,630

   d. 12,602,123,297,335,631

   e. 12,602,123,297,335,632

(over)

7–10: Miscellaneous Problems (6 points each). On your answer sheet, write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

7. When 2019 is written as the sum of two consecutive positive integers, what is the value of the lesser integer?

8. The sum of three numbers equals 275. The first number is 150% of the sum of the other two numbers. What is the value of the first number?

9. What integer is closest to the value of $\sqrt{2019}$?

10. This summer, Paul rode his bike one mile on June 1st, three miles on June 2nd, five miles on June 3rd, and on each succeeding day of June he rode two more miles than he had on the previous day. How many total miles did Paul ride his bike during the thirty days of June?

11–14: Miscellaneous Problems (10 points each). On your answer sheet, write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

11. If $x^2 = 2019$, then what is the exact value of $\dfrac{x^3 - 3x^2 - 2x + 6}{x - 3}$?

12. Suppose that the positive odd integers less than 200 are written in order without using commas or spaces.

$$13579111315171921232527293 1 \cdots 17918118318518718919119319519 7199$$

How many of the written digits are even?

13. How many two-digit prime numbers are there such that the numbers with the digits interchanged are also prime?

14. Determine the <u>sum</u> of the real solutions to $\big(x-3\big)^{2x-6} = 1$.

Student's Answer Sheet **Category II (Algebra, No Calculators)**

Name: _____ Email: _____
PRINT: Last First OPTIONAL

School: _____ High School Graduation Year: _____

I am an alternate for this meet: Yes ☐ No ☐

1–6: Multiple Choice (6 points each). Mark **X** *in the box for the one best choice.*

	a	b	c	d	e
1.	☐	☐	☐	☐	☐
2.	☐	☐	☐	☐	☐
3.	☐	☐	☐	☐	☐
4.	☐	☐	☐	☐	☐
5.	☐	☐	☐	☐	☐
6.	☐	☐	☐	☐	☐

7–10: Miscellaneous Problems (6 points each). Write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

7. _____ *lesser integer* | 6 | ☐

8. _____ *first number* | 6 | ☐

9. _____ *closest integer* | 6 | ☐

10. _____ miles in total | 6 | ☐

11–14: Miscellaneous Problems (10 points each). Write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

11. _____ *exact value* | 10 | ☐

12. _____ even digits | 10 | ☐

13. _____ *number of primes* | 10 | ☐

14. _____ *sum of real solutions* | 10 | ☐

FOR GRADING USE ONLY

#1–6: _____ correct × 6 = _____

#7–10: ..... total = _____

#11–14: ..... total = _____

TOTAL SCORE ☐