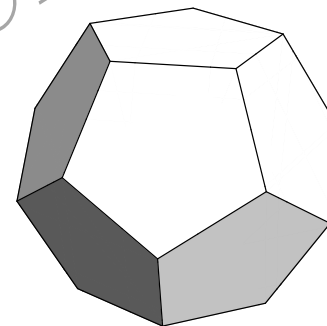# INSTRUCTIONS

1. You will have 55 minutes to complete the exam. A 5-minute warning will be given after 50 minutes.

2. On the answer sheet provided, PRINT your name and school name where indicated, and optionally print your email address and high school graduation year. Mark the proper box to indicate whether you are an alternate for this meet. All answers are to be transferred to the answer sheet, which will be the only page collected. The exam itself is yours to keep and may be marked in any way you wish.

3. Answers will be graded all right or all wrong, except that partial credit may be given for those problems, if any, marked (P). Unless otherwise specified, all answers should be exact and written in simplest form.

4. No calculators may be used. Cell phones and music players are to be turned off and stowed away out of sight.

CENTRAL WISCONSIN

Unplugged

Mathematics League

Advanced Mathematics, No Calculators

Category III

Meet I   November 20, 2019

CENTRAL WISCONSIN MATHEMATICS LEAGUE

Category III   (Advanced Mathematics, No Calculators)

> **Unless otherwise noted, all constants and variables represent real numbers.**
> **Unless otherwise specified, all answers should be exact and written in simplest form.**
> **If a problem has no solutions, write "no solution" in the blank on your answer sheet.**

Miscellaneous Problems (point values as indicated). On your answer sheet, circle the letter of the one <u>best</u> choice or write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

1. [10 points](P)   *True/False: On your answer sheet, circle* **T** *for each of the following statements that is always true; circle* **F** *for each statement that is not always true.*

   (a) $\dfrac{31 \cdot 32 \cdots 99 \cdot 100}{1 \cdot 2 \cdots 70 \cdot 71}$ is an integer.

   (b) $2^{30} < 3^{20}$

   (c) $\sqrt[3]{3\sqrt{2}} < \sqrt[6]{7}$

   (d) There are a total of 6 possible remainders when an integer is divided by 7.

   (e) $2019^2 + 2 \cdot 2019 + 2$ can be expressed as a product of two consecutive positive integers.

2. [10 points]   Determine the vertex (x, y) for the parabola $y = x^2 + 6x + 5$.

3. [10 points]   Simplify $\dfrac{\left(5 + 2\sqrt{6}\right)\left(\sqrt{3} - \sqrt{2}\right)}{\left(\sqrt{2} + \sqrt{3}\right)}$.

4. [10 points]   Simplify $\dfrac{6^n + 3^n + 2^{n+1} + 2}{2^n + 1}$.

5. [10 points]   Using the coordinate axes on your answer sheet, sketch a complete graph of the function $g(x) = \dfrac{(x-1)^2 + (x-1)}{(x-1)}$. *As part of your sketch, draw a point on the graph and label it with its coordinates.*

6. [10 points]   Given the function $f(x) = \dfrac{5}{x-1}$, determine and simplify $2f(2x) + 2$.

7. [10 points]   Solve the equation $\sqrt{x + 2019} + \sqrt[4]{x + 2019} = 6$.

8. [10 points]   Find the remainder when 3^{2019} is divided by 5.

9. [10 points]   If $a > 2019$, which of the following expressions is equal to

$$a^{1/2} - \frac{a - a^{-2}}{a^{1/2} - a^{-1/2}} + \frac{1 + a^{-2}}{a^{1/2} + a^{-1/2}} + \frac{2}{a^{3/2}}?$$

a. $\dfrac{a^{1/2}}{a^2(a+1)}$

b. $\dfrac{2a^{1/2}}{a^2(a+1)}$

c. $\dfrac{2a^{1/2}}{a^2}$

d. $\dfrac{a^{1/2}}{a^2}$

e. $\dfrac{4a^{1/2}}{a^2(1 - a^2)}$

10. [10 points]   Find the exact value of

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{9 \cdot 10}.$$

Name: _____   Email: _____
          PRINT: Last   First   OPTIONAL

School: _____   High School Graduation Year: _____

I am an alternate for this meet: Yes ☐   No ☐
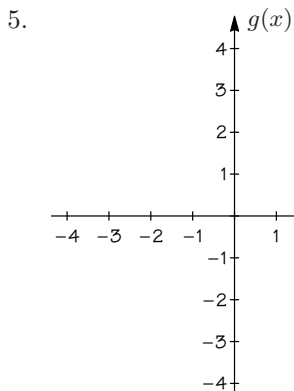
Miscellaneous Problems (point values as indicated). Circle the letter of the one <u>best</u> choice or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

1. (a)   **T**   **F**
    (b)   **T**   **F**
    (c)   **T**   **F**
    (d)   **T**   **F**
    (e)   **T**   **F**

| 10P | |

2. vertex $(x, y) = \left(\underline{\hspace{2cm}} , \underline{\hspace{2cm}}\right)$

| 10 | |

3. _____

| 10 | |

4. _____

| 10 | |

5.

| 10 | |

6. $2f(2x) + 2 = $ _____

| 10 | |

7. _____

| 10 | |

8. _____

| 10 | |

9. *Circle one:*   **a**   **b**   **c**   **d**   **e**

| 10 | |

10. _____

| 10 | |

TOTAL SCORE [ ]