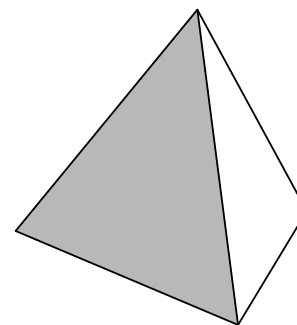


## INSTRUCTIONS

1. You will have 55 minutes to complete the exam. A 5-minute warning will be given after 50 minutes.
2. On the answer sheet provided, PRINT your name and school name where indicated, and optionally print your email address and high school graduation year. Mark the proper boxes to indicate whether you are an alternate for this meet and whether you participated in a previous meet. All answers are to be transferred to the answer sheet, which will be the only page collected. The exam itself is yours to keep and may be marked in any way you wish.
3. Answers will be graded all right or all wrong, except that partial credit may be given for those problems, if any, marked (P). Unless otherwise specified, all answers should be exact and written in simplest form.
4. No calculators may be used. Cell phones and music players are to be turned off and stowed away out of sight.

CENTRAL WISCONSIN

## *Mathematics League*



## Geometry, No Calculators Category I

Meet II

January 29, 2020

Sponsors: Ameriprise Financial Services—Niemeyer, Ledvina and Associates • Church Mutual Insurance • Delta Dental Plans Association • Regnier Consulting Group • Sentry Insurance • Skyward • University of Wisconsin-Stevens Point



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University of Wisconsin  
**Stevens Point**

# CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet II

January 29, 2020

Category I (Geometry, No Calculators)

**Unless otherwise noted, each question below refers to Euclidean geometry.  
Unless otherwise specified, all answers should be exact and written in simplest form.**

1–8: Multiple Choice (5 points each). On your answer sheet, circle the letter of the one best choice.

1. If  $\triangle JKL \cong \triangle MNO$ , which statement must be true?

- a.  $\angle KLJ \cong \angle NMO$
- b.  $\angle KJL \cong \angle MON$
- c.  $\overline{JL} \cong \overline{MO}$
- d.  $\overline{JK} \cong \overline{ON}$
- e. The points on  $\triangle JKL$  and  $\triangle MNO$  are the same.

2. In  $\triangle DEF$ ,  $m\angle D = (3x + 5)^\circ$ ,  $m\angle E = (4x - 15)^\circ$ , and  $m\angle F = (2x + 10)^\circ$ . Which statement must be true?

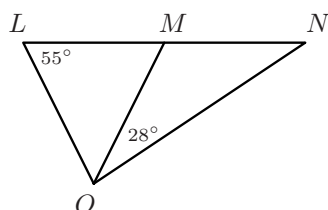
- a.  $DF = EF$
- b.  $DE = EF$
- c.  $m\angle E = m\angle F$
- d.  $m\angle D = m\angle F$
- e.  $\triangle DEF$  is equilateral

3. In parallelogram  $ABCD$ , diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect at  $E$ . Which statement must be true?

- a.  $\overline{AC} \cong \overline{DB}$
- b.  $\angle AEB \cong \angle BEC$
- c.  $\angle ABD \cong \angle CBD$
- d.  $\triangle AED \cong \triangle CEB$
- e.  $\triangle DCE \cong \triangle BCE$

4. As shown below, point  $M$  lies on  $\overline{LN}$  and  $\triangle LMO$  is isosceles with  $LO = MO$ . If  $m\angle L = 55^\circ$  and  $m\angle NOM = 28^\circ$ , what is  $m\angle N$ ?

- a.  $27^\circ$
- b.  $28^\circ$
- c.  $30^\circ$
- d.  $42^\circ$
- e.  $70^\circ$

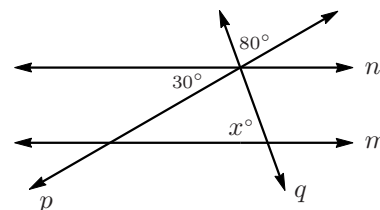


5. Distinct points  $A, B, C$ , and  $D$  are arranged in order along a line such that  $\overline{AC} \cong \overline{BD}$ . Using this information, it can be proven that:

- a.  $BC = AB$
- b.  $BC = CD$
- c.  $AB = CD$
- d.  $AD - BC = CD$
- e.  $AC + BD = AD$

6. As shown below, lines  $m$  and  $n$  are cut by transversals  $p$  and  $q$  that intersect at a point on  $n$ . What value of  $x$  makes the lines  $m$  and  $n$  parallel?

- a. 110
- b. 80
- c. 70
- d. 60
- e. 50



7. The angles of  $\triangle ABC$  are in the ratio of 8 : 3 : 4. What is the measure of the smallest angle?

- a.  $12^\circ$
- b.  $24^\circ$
- c.  $36^\circ$
- d.  $48^\circ$
- e.  $72^\circ$

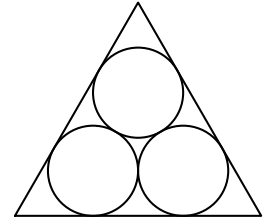
8. Which convex quadrilateral has diagonals that always bisect its angles and also bisect each other?

- a. isosceles trapezoid
- b. kite
- c. parallelogram
- d. rectangle
- e. rhombus

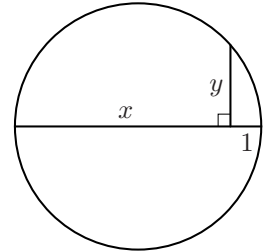
(over)

9–14: *Miscellaneous Problems (10 points each). On your answer sheet, write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.*

9. [10 points] An equilateral triangle and three unit circles are mutually tangent as shown. Determine the area of the equilateral triangle.

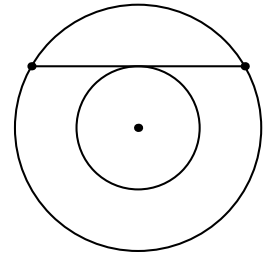


10. [10 points] A line segment  $y$  units long is drawn from a point on a circle to a diameter of the circle so that it is perpendicular to the diameter and divides the diameter into line segments with lengths  $x$  and 1 as shown. Determine the exact value of  $y$  in terms of  $x$ .



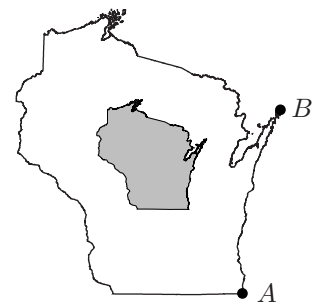
11. [10 points] Consider a string that encircles the globe at the equator and lies snug along the surface of a smooth, spherical Earth. If the string is shortened by 1 meter, thereby causing the string to sink to a uniform depth below the surface around the globe, how far below the surface will the shortened string be in meters?

12. [10 points] The two circles shown are concentric and the length of the chord tangent to the inner circle is 30 mm. Determine the area between the two circles. *Give the exact answer in square millimeters.*



13. [10 points] What is the exact angle between the hour and minute hands of a clock that reads 10:50?

14. [10 points] For the larger image of Wisconsin shown, the length from  $A$  to  $B$  is 2.5 cm. The corresponding length for the proportionately-sized, smaller Wisconsin shown is 1 cm. If the area of the smaller Wisconsin is  $0.2 \text{ in}^2$ , determine the area inside the larger Wisconsin and outside the smaller Wisconsin. *Give the exact answer in square inches.*



Name: \_\_\_\_\_  
PRINT: Last First

Email: \_\_\_\_\_  
OPTIONAL

School: \_\_\_\_\_

High School Graduation Year: \_\_\_\_\_

I am an alternate for this meet: Yes  No

I participated in Meet I: Yes  No

*Miscellaneous Problems (point values as indicated). Circle the letter of the one best choice or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.*

1. Circle one:     **a**     **b**     **c**     **d**     **e**
2. Circle one:     **a**     **b**     **c**     **d**     **e**
3. Circle one:     **a**     **b**     **c**     **d**     **e**
4. Circle one:     **a**     **b**     **c**     **d**     **e**
5. Circle one:     **a**     **b**     **c**     **d**     **e**
6. Circle one:     **a**     **b**     **c**     **d**     **e**
7. Circle one:     **a**     **b**     **c**     **d**     **e**
8. Circle one:     **a**     **b**     **c**     **d**     **e**

9. \_\_\_\_\_ square units

10.  $y =$  \_\_\_\_\_  
in terms of  $x$

11. \_\_\_\_\_ meters

12. \_\_\_\_\_ square millimeters

13. \_\_\_\_\_ degrees

14. \_\_\_\_\_ square inches

5	
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**TOTAL SCORE**