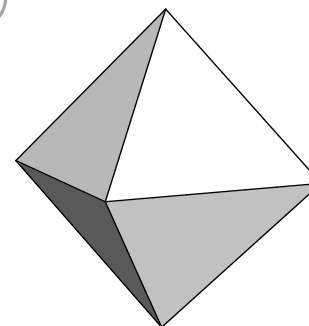


INSTRUCTIONS

1. You will have 55 minutes to complete the exam. A 5-minute warning will be given after 50 minutes.
2. On the answer sheet provided, PRINT your name and school name where indicated, and optionally print your email address and high school graduation year. Mark the proper boxes to indicate whether you are an alternate for this meet and whether you participated in a previous meet. All answers are to be transferred to the answer sheet, which will be the only page collected. The exam itself is yours to keep and may be marked in any way you wish.
3. Answers will be graded all right or all wrong, except that partial credit may be given for those problems, if any, marked (P). Unless otherwise specified, all answers should be exact and written in simplest form.
4. No calculators may be used. Cell phones and music players are to be turned off and stowed away out of sight.

CENTRAL WISCONSIN

Mathematics League



Algebra, No Calculators Category II

Meet II

January 29, 2020

Sponsors: Ameriprise Financial Services—Niemeyer, Ledvina and Associates • Church Mutual Insurance • Delta Dental Plans Association • Regnier Consulting Group • Sentry Insurance • Skyward • University of Wisconsin-Stevens Point



Listening. | Learning. | Leading.®



University of Wisconsin
Stevens Point

CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet II

January 29, 2020

Category II (Algebra, No Calculators)

Unless otherwise noted, all constants and variables represent real numbers.
Unless otherwise specified, all answers should be exact and written in simplest form.

1–6: Multiple Choice (6 points each). On your answer sheet, mark **X** in the box for the one best choice.

1. How many integers between 100 and 1000 have only odd digits?
 - a. 100
 - b. 125
 - c. 499
 - d. 501
 - e. 729
2. Solve the inequality: $|x + 5| < 1$
 - a. $x < -4$ or $x > 6$
 - b. $4 < x < 6$
 - c. $x < -6$ or $x > -4$
 - d. $-6 < x < -4$
 - e. $x < 4$ or $x > 6$
3. The polynomial $36x^4 - 36x^3 - x^2 + 9x - 2$ has real roots $r_1 < r_2 < r_3 < r_4$. The sum $r_1 + r_2 + r_3 + r_4$ equals
 - a. 1
 - b. 4
 - c. 9
 - d. 18
 - e. 36
4. Given that the system of equations $y = x^2$ and $ax + by = c$ has exactly two real solutions (x_1, y_1) and (x_2, y_2) , which of the following must always be true?
 - a. $b^2 - 4ac > 0$
 - b. $a^2 + 4bc > 0$
 - c. $a^2 + 4bc < 0$
 - d. $a^2 - 4bc < 0$
 - e. $b^2 + 4ac < 0$
5. What is the minimum value of $(x - 1)^2 + (x - 3)^2 + (x - 5)^2$?
 - a. 0
 - b. 8
 - c. 10
 - d. 13
 - e. 38
6. Let a_n be the sequence of integers defined by $a_1 = 2$, $a_2 = 7$, $a_3 = 4$, $a_4 = 8$, and in general a_{n+2} equals the ones digit of the product $a_{n+1} \cdot a_n$. What is the value of a_{2020} ?
 - a. 2
 - b. 7
 - c. 4
 - d. 8
 - e. 6

(over)

7–10: *Miscellaneous Problems (6 points each). On your answer sheet, write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.*

- When 2020 is written as the sum of five consecutive integers, what is the value of the least integer?
- What is the greatest positive integer less than 1000 that is both a perfect square and a perfect cube?
- What is the smallest positive integer k so that the product

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot k$$

is a perfect square?

- What is the greatest real root of $2x^4 + 3x^3 + 4x^2 + x - 2$?

11–14: *Miscellaneous Problems (10 points each). On your answer sheet, write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.*

- A mathematics contest has 20 problems. Students gain 4 points for each correct answer and lose 2 points for each incorrect answer. Jordan answered every question and received a score of 68 points. How many correct answers did Jordan have?
- If $x^2 + y^2 - 4x + 2xy - 4y = 21$, then what is the least value of $x + y$?
- Sinan, Dale, and Terry started sharing algebra problems on the social media site *PointerGram*. Sinan posted 3^{x-1} problems, Dale posted 3^x problems, and Terry posted 3^{x+1} problems. None of the 351 problems they all posted were the same. How many problems did Sinan post?
- When $x^2 + bx + 3$ is divided by $x - 1$ the remainder is twice the remainder when $x^2 + bx + 3$ is divided by $x + 1$. What is the value of b ?

Name: _____
PRINT: Last First

Email: _____
OPTIONAL

School: _____

High School Graduation Year: _____

I am an alternate for this meet: Yes No

I participated in Meet I: Yes No

1–6: Multiple Choice (6 points each). Mark **X** in the box for the one best choice.

	a	b	c	d	e
1.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

7–10: Miscellaneous Problems (6 points each). Write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

7.	<i>least integer</i>	6	
8.		6	
9.	<i>smallest k</i>	6	
10.		6	

11–14: Miscellaneous Problems (10 points each). Write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

11.	_____ correct answers	10	
12.	_____	10	
13.	_____ problems	10	
14.	$b =$ _____	10	

FOR GRADING USE ONLY

#1–6: _____ correct \times 6 = _____

#7–10: total = _____

#11–14: total = _____

TOTAL SCORE

<div style="border: 1px solid black; width: 100%; height: 100%;"></div>
