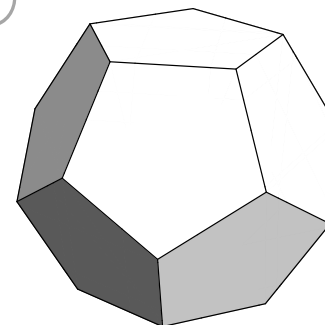


INSTRUCTIONS

1. You will have 55 minutes to complete the exam. A 5-minute warning will be given after 50 minutes.
2. On the answer sheet provided, PRINT your name and school name where indicated, and optionally print your email address and high school graduation year. Mark the proper boxes to indicate whether you are an alternate for this meet and whether you participated in a previous meet. All answers are to be transferred to the answer sheet, which will be the only page collected. The exam itself is yours to keep and may be marked in any way you wish.
3. Answers will be graded all right or all wrong, except that partial credit may be given for those problems, if any, marked (P). Unless otherwise specified, all answers should be exact and written in simplest form.
4. No calculators may be used. Cell phones and music players are to be turned off and stowed away out of sight.

CENTRAL WISCONSIN

Mathematics League



Advanced Mathematics, No Calculators Category III

Meet II

January 29, 2020

Sponsors: Ameriprise Financial Services—Niemeyer, Ledvina and Associates • Church Mutual Insurance • Delta Dental Plans Association • Regnier Consulting Group • Sentry Insurance • Skyward • University of Wisconsin-Stevens Point



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University of Wisconsin
Stevens Point

CENTRAL WISCONSIN MATHEMATICS LEAGUE

Meet II

January 29, 2020

Category III (Advanced Mathematics, No Calculators)

**Unless otherwise noted, all constants and variables represent real numbers.
Unless otherwise specified, all answers should be exact and written in simplest form.
If a problem has no solutions, write “no solution” in the blank on your answer sheet.**

Miscellaneous Problems (point values as indicated). On your answer sheet, circle the letter of the one best choice or write your answer in the blank(s) provided. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

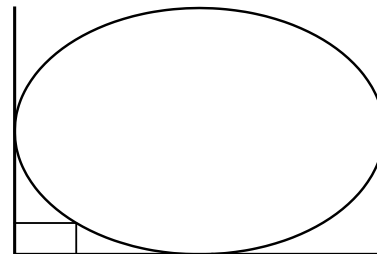
1. [10 points](P) A check is written for x dollars and y cents, where x and y are both two-digit numbers. In error, the check is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$9.90.

*True/False: On your answer sheet, circle **T** for each of the following statements that is true for the given check-cashing scenario; circle **F** for each statement that is not true.*

- (a) y is greater than x .
- (b) y can equal $2x$.
- (c) The sum of the digits of the correct amount must be divisible by 9.
- (d) x must be less than 90.
- (e) Both x and y must be divisible by 5.
2. [10 points] Given the function $f(x) = \frac{5x}{(x-1)^2 - 1}$, determine and simplify $\frac{1}{5}[f(x+1) - f(1)]$ when $x \neq \pm 1$.
3. [10 points] Using the coordinate axes on your answer sheet, sketch a complete graph of the function $g(x) = ||x - 1| - 1|$. As part of your sketch, draw and label the points $(0, g(0))$ and $(2, g(2))$ as **A** and **B**, respectively.
4. [10 points] How many integers n satisfy $n^4 + 11n < 11n^3 + n^2$?
5. [10 points] How many integers between 99 and 999 do not have any of the digits 4, 5, 6, 7, 8 nor 9?

(over)

6. [10 points] Let a , b and c be the three real solutions for the equation $x^3 - 3x^2 + x + 1 = 0$. Determine $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$.
7. [10 points] Chris is thinking about joining a club at UW-Stevens Point. Chris knows two members of the club who happen to be math majors. Chris also knows that the club membership is more than 91% non-math majors. What is the minimum number of members in this club?
8. [10 points] How many ordered pairs (x, y) of integers are solutions to $x^2 + y^2 = 6x + 1$?
9. [10 points] An elliptical table with major axis of length 12 meters is pushed to the corner of a rectangular room so that the major and minor axes are parallel to the walls of the room (see figure). There is exactly enough room left in the corner for a 1-meter by $(6 - \sqrt{20})$ -meter rectangular table. This rectangular table is oriented so that the side with length 1 meter is parallel to the minor axis of the elliptical table. Determine the exact length of the minor axis of the elliptical table.
10. [10 points] Determine the positive integers n that make $n^2 - 65$ a perfect square.



Name: _____
PRINT: Last First

Email: _____
OPTIONAL

School: _____

High School Graduation Year: _____

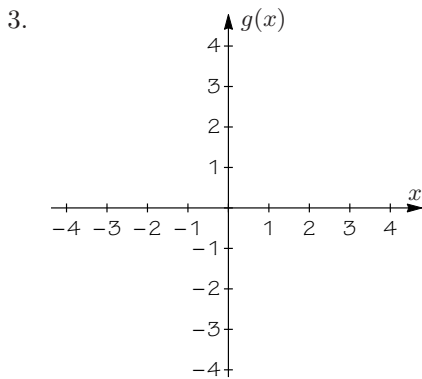
I am an alternate for this meet: Yes No

I participated in Meet I: Yes No

Miscellaneous Problems (point values as indicated). Circle the letter of the one best choice or write your answer in the blank(s) provided; the boxes at the right are for grading use only. (P) means that partial credit may be given. Additional instructions can be found in the box at the top of the exam.

1. (a) **T** **F**
- (b) **T** **F**
- (c) **T** **F**
- (d) **T** **F**
- (e) **T** **F**

2. _____



4. _____ integers n

5. _____ integers

6. _____

7. _____

8. _____ pairs (x, y)

9. _____ meters

10. _____

10P	
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TOTAL SCORE